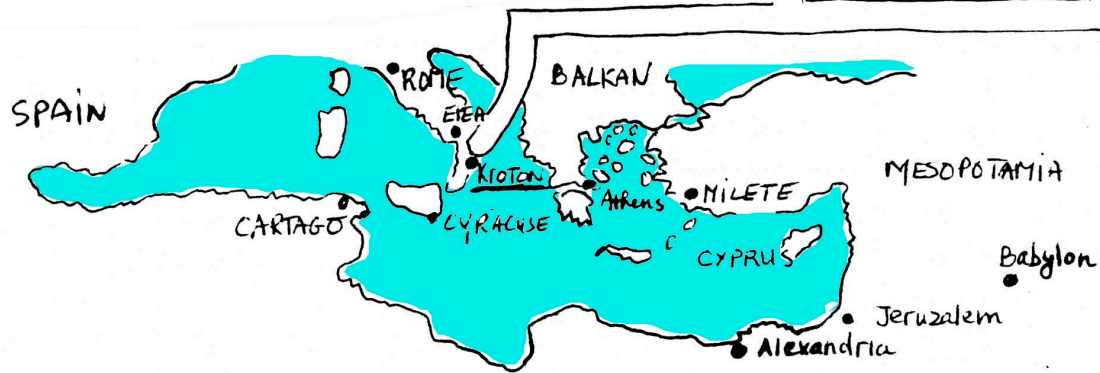
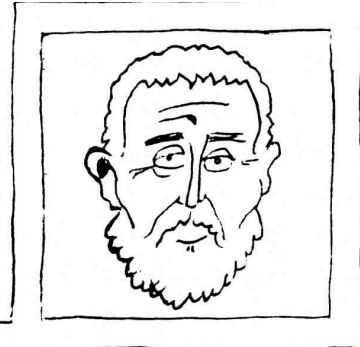


Pythagoras, Scientist and Globetrotter.

About 2500 years ago in the ancient city of Kroton at the Mediterranean sea lived a scientist called Pythagoras.

Pythagoras wasn't exactly your typical scientist spending his life in dusty rooms with scrolls, contemplating difficult texts or math problems. On the contrary, he was a very active person, travelling a lot during his life-time. He visited Alexandria and Babylon, being quite a travel in his times. Some historians even think he has been in India.

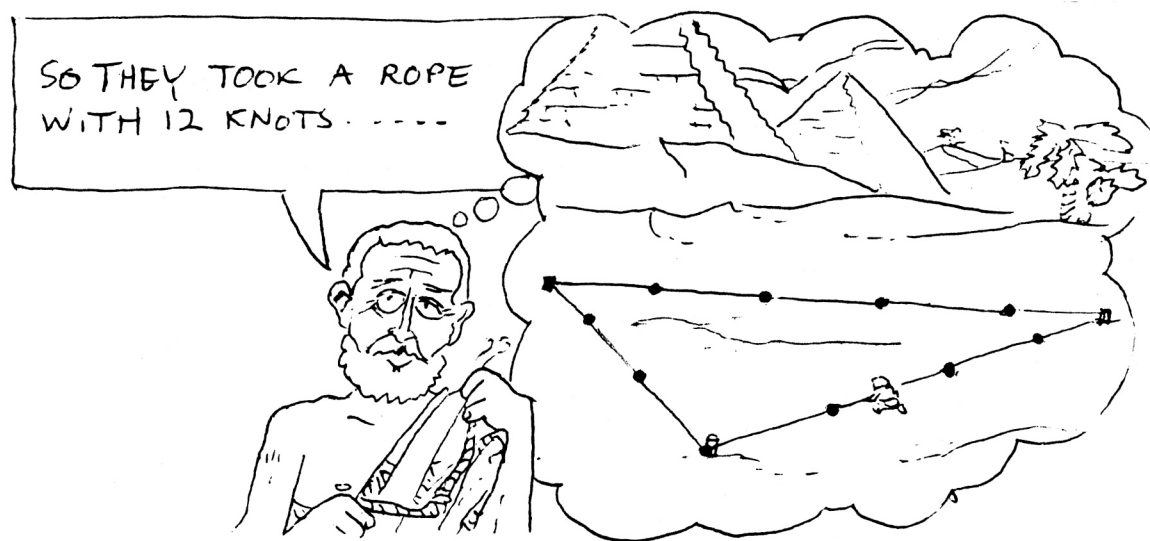


Returning from his travels he shared with anyone who was willing to listen what he had experienced in these 'far away' eastern countries. he held lectures in Music, Astronomy and Geometry, because he learned a lot about these subjects from the Egyptians and the Babylonians.



ROPE STRETCHERS IN EGYPT.

During one of these educational hours Pythagoras showed his friends how Egyptian contractors could make right angles very accurately.



1. *Why is it Important to be able to make right angle so accurately in constructing a pyramid for instance? Do you have any idea how modern contractors do this ?*
2. *Get a piece of rope and mark 12 equal lengths on the rope. (The Egyptians used knots for this division, but if this seems unhandy to you, then you could use a felttipwriter or colored tape to mark these divisions.) Tie the ends together such that the closed rope has exactly 12 marks on it.*
3. *With this rope you can make different 12-knot triangles. These triangles must have a knot (or a mark) on each vertex. Three people are needed to hold the vertices and keep the rope taut. How many distinctly different triangles can you make? Try this out.*
4. *On a piece of graph paper draw all your different 12-knot triangles as accurately as possible. Use the distance between two grid points as the distance between two knots (marks) on the rope.*
5. *Which triangle(s) that you constructed was (were) used by the Egyptian rope stretchers? Explain why.*
6. *One of the triangles you constructed has some special characteristics that allow you to determine the size of each of its angles without doing any measuring. Find that triangle and identify its angle measures. Explain your thinking.*
7. *Finally, what conjecture(s) can you make about the perimeter of each constructed triangle? Can you make the same conjecture(s) for the area of each constructed triangle? Explain your thinking.*

THE SECRET OF THE ROPE STRETCHERS.

Pythagoras taught his friends how they could make a right triangle with the 12-knot rope. but his friends were very critical and didn't take his story for granted.

"Can't you do it with less knots Can you prove it That angle that you call right might be $89 \frac{1}{2}$ degrees. Who can tell?" "

Pythagoras wasn't bogged down by this. He had an answer to everything, like the rest of this story will show us. Pythagoras said, "Of course, there are lots of triangles you can make with ropes containing less than 12 knots; in fact a total of 15 different ways, but there is not a single right triangle among them!"

8. You will receive a handout with two triangles constructed from ropes with less than 12 knots. Use a straight edge and a compass to construct the other 13 triangles. Verify thaty there is not a single right triangle among the 15 triangles.

Right triangles are apparently quite rare. At least this seems to be the case when we wish the sides of these triangles to measure a whole number of marks/knots. Pythagoras knew some more cases besides the 12-knot rope. He made the following list for his friends:

Number of knots	Length of the sides for right triangle
12	3 , 4 , 5
24	6 , 8 , 10
30	5 , 12 , 13
36	9 , 12 , 15
40	8 , 15 , 17

9. Do you notice any patterns in this table? Please explain why you think that pattern exists.

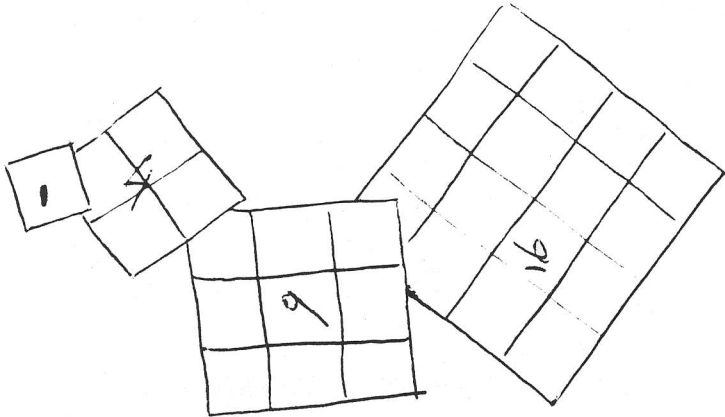
It was still not clear how he got these numbers. Well...you better get to know the secret of the rope stretchers!

PYTHAGORAS AND THE TILER OF THE PHARAO.

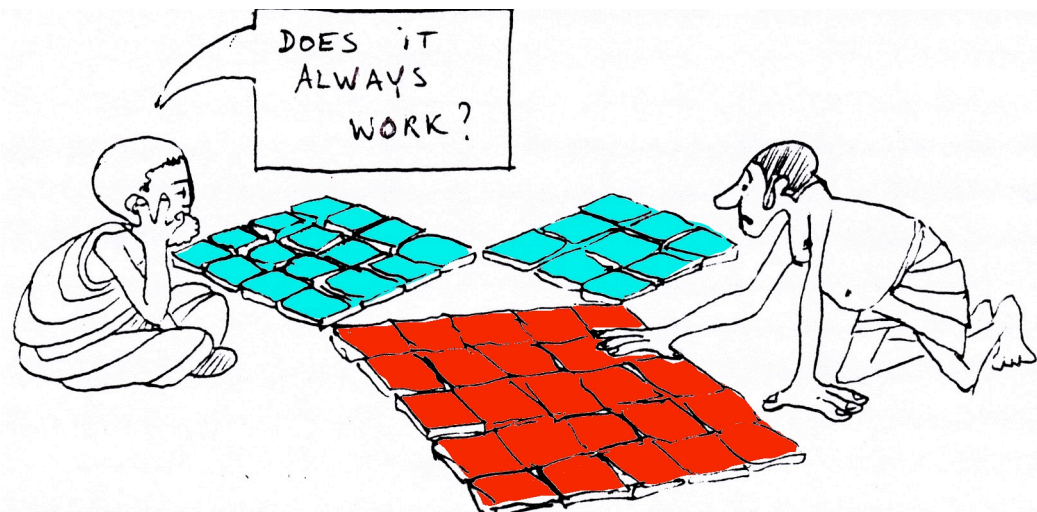
Filolaios, a good friend of Pythagoras, wanted to know how Pythagoras had discovered the secret of the rope stretchers. Pythagoras told him the following story:

"You have to know that a few days after my encounter with the rope stretchers I was received by the Pharao of Egypt. I gathered all my courage and asked the Pharao to tell me the secret. The Pharao called for his tiler and asked him to make 2500 tiles of which 1250 as red as ivory and 1250 as blue as the sky. All tiles were squares. When the tiler returned with the tiles, the Pharao told him to construct as many perfect squares as possible with the red tiles and do the same with the blue tiles."

10. Determine all the different perfect squares the tiler could make using up to 1250 red and also up to 1250 blue squares. Then get two differently colored pieces of graphpaper and work with a partner to cut out as many perfect squares as possible starting at a square of size 1, then 4, then 9, and so on. Write the area on each of the perfect squares.



Pythagoras continues: "Once the tiler was done I asked the Pharaoh for the relation between rope stretchers and the tiler and the Pharaoh simply replied to watch how his tiler makes a triangle with the tile-squares."



11. Pick any three perfect squares and form a triangle with them just like the tiler in the illustration above. Can you always form a triangle this way with any three perfect squares that you pick? When does it certainly not work? Experiment with the perfect squares you cut out and develop a conjecture. Explain why your conjecture is true.

Pythagoras then explains: "And the Pharaoh assigned his tiler to create a sequence of triangles for which the largest side has a red perfect square on it and the two smaller sides a blue perfect square. One tiler counted the number of small blue tiles, another the number of small red tiles and a third used a protractor to find the measure of the largest angle. If that angle was 90 degrees, they added it to the table that I showed you earlier."

12. *The tiler used two blue perfect squares of sizes 25 and 64. If the triangle he constructed had all three angles acute, then what was the size of the red perfect square?*

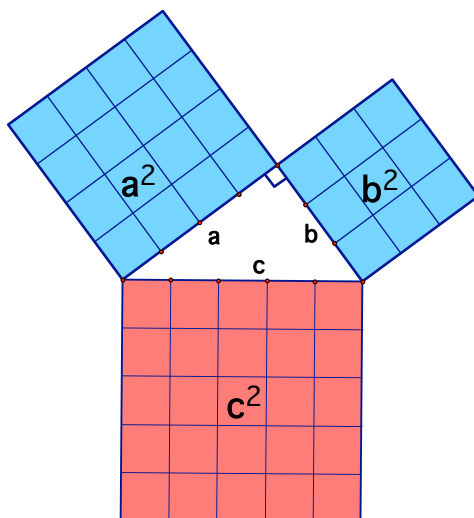
13. *To make another triangle the tiler used two blue perfect squares of sizes 100 and 81. This time he made an obtuse triangle. What was the size of the red perfect square?*

14. *With a partner construct several triangles using two blue and one red perfect square. Copy the table below and record the sum of the sizes of the two blue perfect squares, the size of the red perfect square and in the third column identify whether the triangle you constructed is acute, right, or obtuse. Then study your table for data and make a conjecture. Explain how you discovered your conjecture.*

Triangle	Sum of the sizes of Blue Perfect Squares	Size of the Red Perfect Square	Type of Triangle (acute, right, obtuse)
1			
2			
3			
4			
5			
6			
7			

15. *Based on your conjecture, predict what type of triangle will be formed when using blue perfect squares of size 100 and 225, and a red perfect square of 400. Explain your reasoning.*

16. *Describe in your own words what the relationship is between how the rope stretchers made a right triangle and how the tilers did this. Express your conclusions about this relationship in an algebraic formula. The illustration below may assist you.*



This text was translated by Cornelis de Groot, Ph.D., from a Dutch book: De Stelling Van Pythagoras (pp. 2-7). It was authored and published by Instituut voor de Ontwikkeling van het Wiskunde Onderwijs (IOWO), now the Freudenthal Institute at the University of Utrecht, the Netherlands. Approximate date is between 1975 and 1980.